Agenda

Reading 32: The Term Structure and Interest Rate Dynamics

- Describe relationships among spot rates, forward rates, yield to maturity, expected and realized returns on bonds, and the shape of the yield curve
- Describe the forward pricing and forward rate models and calculate forward and spot prices and rates using those models
- Describe how zero-coupon rates (spot rates) may be obtained from the par curve by bootstrapping
- Describe the assumptions concerning the evolution of spot rates in relation to forward rates implicit in active bond portfolio management
- Describe the strategy of riding the yield curve
- Explain the swap rate curve, its purpose and uses in valuation.
- Calculate and interpret the swap spread for a given maturity
- Describe the Z-spread, TED spread and Libor–OIS spreads
- Explain theories of the term structure of interest rates.
- Describe modern term structure models and how they are used
- Explain how a bond's exposure to factors driving the yield curve can be measured and how these exposures can be used to manage yield curve risks
- Explain maturity structure of yield volatilities and its impact on bond price.

Spot Rate

Spot rate is a rate of interest today on a zero coupon bond.

- Spot rate is the **discount rate** that equates $1 to be received in the future with the value of the $1 today.

Example: $1 to be received in 1 years time is trading for $0.962 today

$$P_T = \frac{1}{(1+r_T)^T} \quad \Rightarrow \quad 0.962 = \frac{\$1}{(1+r)^1} \quad \Rightarrow \quad r = 3.95\%$$

- Spot rates (or discount rates) when computed for a range of maturities is called the discount function, spot yield curve (or spot curve).
- The spot curve represents annualized return on an option free, default free bond with single (bullet) payment at maturity (zero coupon bond).
- The spot curve simply provides information about time value of money for all points along the maturity.
- Spot curve does not make any 'reinvestment rate' assumption.
- The shape and level of the yield curve is dynamic i.e. constantly changing.

Forward Rates

Forward rate is the rate of interest set today for a loan or zero coupon security to be issued at a future date.

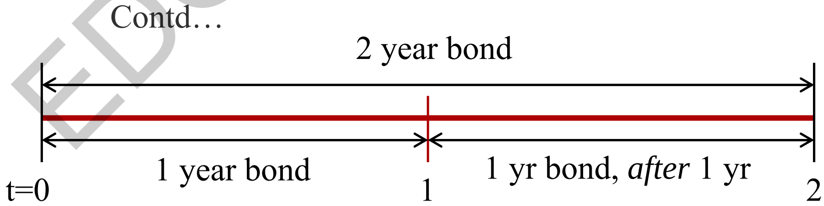
- The term structure of forward rates for a loan made on a specific initiation date is called the forward curve.
- Forward rates and the forward curve are *not 'observed'*, rather they must be *derived* from the spot rate curve.
- To derive the forward rates we use the **forward pricing model** which is based on the 'no arbitrage' principle.
- The no arbitrage principle (law of one price) states that, any two tradable securities producing identical cash flows must have the same price.

Example: To avoid potential arbitrage, the two portfolios below must have the same ending value and must incur the same borrowing cost:

1. Borrow $100 in a two year zero coupon bond offering 5% p.a.
2. Borrow $100 in a one year zero coupon bond offering 3.5% per annum, and then refinance the loan (roll it) over into another one-year loan a year later.

Forward Rates

Example: Contd…



- The forward rate is quoted as $F(T^*, T)$.
- Forward rate is the rate T^* periods from now on a T-period zero coupon bond

Forward Rate Model – demonstrates the relationship between spot rates and the forward rates.

$$(1 + S_2)^2 = (1 + S_1)^1 (1 + F(1,1))^1$$

From the previous example, the forward price rate is:

$$(1 + 5\%)^2 = (1 + 3.5\%)[1 + F(1,1)] \Rightarrow \quad F(1,1) = 6.5217\%$$

We can use the forward rate $\mathbf{F(1,1)}$ to determine the price of a forward bond after T* years as follows:

$$\$\,F_{1,1} = \frac{\$1}{(1 + F[1,1])^1} \quad \Rightarrow \quad \text{Solve for } \$\,F_{1,1}$$

Forward Rates

Forward Pricing Model

- The forward *rate* model priced the forward bond by first determining the forward rate and *then* computing the forward bond price.
- Alternatively we can price forward bonds directly using prices of spot bonds using the forward *pricing* model.
- 2 year zero offering 5% would have current price (i.e. discount factor):

$$P_2 = \frac{\$1}{(1+0.05)^2} \quad \Rightarrow \quad P = \$0.90703$$

- 1 year bond offering 3.5% would have current price (i.e. discount factor):

$$P_1 = \frac{\$1}{(1+0.035)^1} \quad \Rightarrow \quad P = \$0.966184$$

- Price of the 1 year bond after 1 year: $\mathbf{P_2 = P_1 \times F_{1,1}}$
  $F_{1,1} = 0.938776 \Rightarrow$ 1 yr bond 1 yr from now will trade at 93.87% of par
- This implies that forward bond price is the ratio of the two discount factors.

$$\frac{\mathbf{P_2}}{\mathbf{P_1}} = \mathbf{F_{1,1}}$$

Example

Q1: A one year zero coupon bond offers 7%. A three year zero coupon bond offers 9%. Calculate:

1) The one-year discount factor P(1)
2) The three year discount factor P(3)
3) Calculate the forward price of a two year bond to be issued in 1 year.
4) Calculate the forward rate F(1,2)

Forward Rates

Interpretation of Forward Rates:

- **Break-even rate** – forward rates are rates that will make an investor indifferent between investing for the full investment horizon (9% for 3 years) OR part of the investment horizon (7% for 1 year) and rolling over the proceeds for the balance of the investment horizon (10% for 2 years).

- Forward rates allow an investor to **lock in** a rate for some future period. Example: A 2 year bond offering 10% interest is equivalent to investing 1 year at 5% and rolling forward the investment for 15.23%. Thus by investing in a 2 yr bond, the investor has locked in the forward rate of 15.23%.

Relationship between Spot Rates and Forward Rates

1. Spot rates are geometric averages of forward rates.

Example: Determine the relationship between a 5 year spot rate S_5 and

a) S_4 and F(4,1)

b) S_3 and F(3, 1) and F(4,1)

c) S_2 and F(2,1) and F(3,1) and F(4,1)

Forward Rates

Relationship between Spot Rates and Forward Rates

2. When the spot rates are rising → forward rates will be higher than the spot rates and the forward curve will lie above the spot rate curve.

- When the spot rates are falling → forward rates will be lower than the spot rates and the forward curve will lie below the spot rate curve.
- When the spot rates are constant → forward rates will equal spot rates and both the forward curve and spot curve will be flat.

Demonstration

Given the spot rates r(1) = 7%, r(2) = 8% and r(3) = 9%. Determine if the forward rate F(2,1) is greater than the long term rate r(3)

$$(1+S_3)^3 = (1+S_2)^2(1+F[2,1])^1$$

$$\frac{(1+S_3)^3}{(1+S_2)^2} = (1+F[2,1]) \quad \Rightarrow \quad \frac{(1+9\%)^3}{(1+8\%)^2} = (1+F[2,1])$$

Since the numerator is larger than the denominator the forward rate will be higher than the spot rate

Forward Rates

Relationship between Spot Rates and Forward Rates

3. If the spot curve is downward sloping, postponing the initiation of the forward rate (eg, replace F(1,1) with F(2,1)) results in progressively lower forward rates and vice versa.

Example: Assume the following spot rates, r(0.5) = 6.30%; r(1) = 7.02% and r(1.5) = 7.52%

i. Determine the forward rate F(0.5,1), is this rate higher or lower than r(1.5)?
ii. If the forward rate initiation is postponed to F(1,0.5); how does this rate compared to r(1.5) and F(0.5,1)?

4. Forward rates do not (and cannot) extend beyond the furthest maturity on the spot yield curve.

Example: if the spot yield curve plots rates until 30 year maturities, forward rates initiated after 2 years will yield a rate for F(2,28).

Forward Rates

Evolution of Forward Prices

- ***Note: Forward rates are also referred to as future spot rates.***
- If the future spot rates turn out exactly as forecasted by the forward rate curve, then the forward prices on bond will remain unchanged.
- **Conclusion:** a *change in the forward price* of a bond results from actual spot rates deviating from spot rates forecasted using forward rates.
- This relationship is useful for active bond investors:
  - ❖ If the *investor believes* that *future spot rates will be lower* than what is observed (or forecasted) today, this *implies that forward rates will also be lower* i.e. forward prices on bonds will be higher in the future.
    Strategy: buy the bond today (buy cheap), wait till the spot rates fall and forward prices rise to sell the bond (sell expensive)
  - ❖ If the *investor believes* that *future spot rates will be higher* than what is observed today, this *implies that forward rates will be higher* too i.e. forward prices on bonds will be lower in the future.
    Strategy: short the bond today (sell expensive), wait till the spot rates rise and forward prices to fall to buy the bond back (buy cheap)

Par Curve – Bootstrapping

- Forward rate are derived from spot rates. Where are spot rate obtained from?

Recall: spot rates (aka. zero rate) are interest rates on zero coupon securities.

- The most common zero coupon security traded in the market is the US T-bill.
- But T-bills have maturities up to 1 yr, providing spot rates for 0-360 days.
- If spot rates are required for periods longer than 1 year investors use the '**government par bond curve**' and a technique called '**bootstrapping**' to obtain longer term spot rates.

- **Government Par Bond Curve** represents the YTM on government coupon paying bonds, priced at par, over various maturities.

→ The par curve is derived from the yields of '***on-the-run***' Treasuries because new securities tend to trade closer to par.

→ The par curve is determined using an approach called 'Bootstrapping'

→ Bootstrapping is a *process of sequentially calculating longer maturity spot rates from par value securities with different maturities.*

Par Curve – Bootstrapping

Example: Given the following information about a six-month zero-coupon bond and 3 bonds paying coupons semiannually, the annual spot rates for the four consecutive six-month periods using bootstrapping is *closest* to

Maturity (years)	Coupon Rate (i.e. Par Rate)	Price	Spot Rates
0.5	5%	100	
1	5.97%	100	
1.5	6.91%	100	
2	7.81%	100	

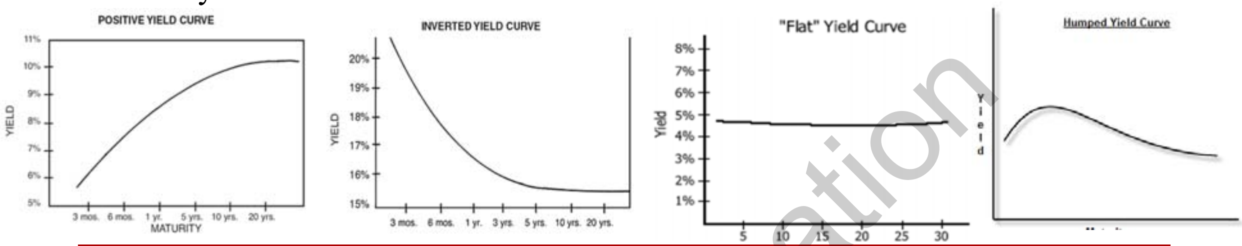
The Yield Curve

Yield Curve

- Refers to the relationship between yields on securities and maturity.

Shapes of the Yield Curve

- Positively sloped (normal) yield curve is where the long term yields are higher than short term yields. ← More common
- Downward sloped (inverted) yield curve is where long term yields are lower than short-term yields ← Uncommon
- Flat curve is where the yields across all maturities is the same ← Common over long maturity ranges
- Humped curve where short- and long-term yields are lower than intermediate term yields ← Rare



Relationship between Spot Rates & Yield to Maturity

Bond Pricing using Spot Rate

- Recall: Spot rates are interest rates on single payment securities (zero coupon bonds). Spot rates vary for each maturity point.
- A coupon bond can be decomposed into a series of zero coupon bonds. Example: A two year 6% annual pay bond comprises of 3 zeros, namely: 1 year bond with face value \$60, 2 year bond with face value \$60 and a 2 year bond with face value \$1000
- To price this bond, we simply discount each zero coupon bond's cash flows *with its respective* spot rate and sum it up.
  Example: Bond price $= \frac{\$60}{(1+S_1)^1} + \frac{\$1060}{(1+S_2)^2}$

Bond Pricing using Yield to Maturity

- Yield to maturity (YTM) is a single discount rate that equates bond cash flows to the price of the bond.

$$964.8 = \frac{\$60}{(1+\text{yield})^1} + \frac{\$1060}{(1+\text{yield})^2}$$

- Hence, YTM is a single rate which is an average of all the spot rates.

Example

Given the spot rates r(1) = 7%, r(2) = 8% and r(3) = 9%.

a) Calculate the price of a two-year 6% annual coupon $1000 face value bond using the spot rates.

b) Using the bond price in (a), calculate the YTM on the bond. How does the YTM compare to r(2)?

c) Calculate the price of a three year 5% annual coupon £100 face value bond.

d) Using the bond price in (c), calculate the YTM on the bond. How does the YTM compare to r(3)?

Yield To Maturity and the Expected Return on the Bond

Q: Does YTM = Expected return on the bond?

Ans: Yes, only if three (very restrictive) assumptions are met:

1) Bond is held to maturity. A bond can still earn a return = YTM if sold before maturity as long as YTM at the time of sale = YTM at the time of purchase.
2) All coupon and principal payments are made in full.
3) All coupons are promptly reinvested at the original YTM. YTM implicitly assumes that the yield curve is flat.

- Only if the above conditions are met will the bond's YTM equal the expected return on the bond.

- YTM provides a poor estimate of expected return on a bond when:

Reinvestment of coupons is not at original YTM rate.
1. Interest rates are volatile
2. Yield curve is steeply upward/downward sloping

Cashflow not received as planned.
3. Significant risk of default

Bond is not held to maturity.
4. Bond has one or more embedded options.

- Thus, the realized (actual) return on a bond will depend on the actual reinvestment rates and yield curve at the end of the bond's holding period.

Relationship between Spot Rates & Forward Rates

Bond Pricing using Forward Rates

- Recall: Spot rates are geometric averages of forward rates, i.e, a spot rate can be decomposed into a series of forward rates.
- If spot rates can be used to price a bond, we can use forward rates to price a bond as well.

Example: Pricing of a 3 year annual pay 6% coupon bond.

$$\text{Bond price} = \frac{\$60}{(1+S_1)^1} + \frac{\$60}{(1+S_2)^2} + \frac{\$1060}{(1+S_3)^3}$$

We can replace $(1+S_2)^2 = (1+S_1)(1+F[1,1])$

$(1+S_3)^3 = (1+S_1)(1+F[1,1])(1+F[2,1])$

$$BP = \frac{\$60}{(1+S_1)^1} + \frac{\$60}{(1+S_1)(1+F[1,1])} + \frac{\$1060}{(1+S_1)(1+F[1,1])(1+F[2,1])}$$

Riding the Yield Curve

Traditionally: Bond investors applied the '*maturity matching*' strategy to purchasing bonds.
<u>Example:</u> Company needs to purchase $100,000 worth of raw materials in 1 yrs time and would like to invest the funds in liquid assets. The company would purchase a $100,000 zero bond at a price of 96% of par maturing in 1yr.

→ This strategy, albeit straightforward may not be a superior investment strategy especially if the yield curve is upward sloping.

Riding the yield curve (aka, rolling down the yield curve)

- Under this strategy the investor purchases bonds with maturity longer than the intended investment horizon.
- In an upward sloping yield curve, longer maturities have higher yields and lower bond prices (cheaper bonds); shorter maturities have lower yields and higher bond prices (expensive bonds).
- **Logic:** Why buy an expensive bond when you can buy the cheap one? Invest your funds in the long maturity bond and then sell the bond to recover cash when you need it.

Riding the Yield Curve

Assumptions

- Riding the yield curve produces higher returns than maturity matching strategy ***if*** the yield curve remains unchanged over the investment horizon i.e. spot rates are assumed to follow the forward curve determined at present.
- Greater the difference between forward rates and spot rates and the longer the maturity of the bond, higher will be the return on the investment.

Demonstration Assume the below spot rates, zero bond prices & forward rates:

r(1) = 9%, 1 year zero = $91.74 r(2) = 10%, 2 year zero = $82.64

r(3) = 11%, 3 year zero = $73.12 F[1,1] = 11.01% F[1,2] = 12.01%

→ Buying a 1 year zero for $91.74 will yield a capital appreciation of $8.26 when the bond redeems at par on maturity date. Company funding $100k purchase would buy #1000 bonds @ $91.74 = $91,740 today to fund their purchase, effectively saving $8,260.

→ Buying a 3 year zero for $73.12 and then selling it for $82.64 will yield a capital gain of $9.52. Company buys #1210 bonds @ $73.12 for $88,475 and sell them for (#1210 x $82.64) ~ $100k after 1 yr, effectively saving $11,525

Swap Curve

Recall:

- Swaps are derivative instruments that involve participants exchanging (swapping) one cash flow stream for another.
- The plain vanilla interest rate swap is agreement to exchange fixed-rate interest payments for floating rate interest payments.
- The fixed-rate interest is called the swap rate, and the principal against which interest is calculated is called the notional principal.
- The floating-rate interest is based on a benchmark interest rate such as the Libor or Euribor or Tibor.
- Libor, Euribor or Tibor are interest rates bank's charge one another for borrowing and lending dollar, euro or yen denominated loans.

Swap Rate Curve (or Swap Curve)

- A graphical plot of the swap rates (fixed-rate interest) across various swap maturities.